# Efficient Algorithms for Learning Revenue-Maximizing Two-Part Tariffs

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# Two-part tariffs

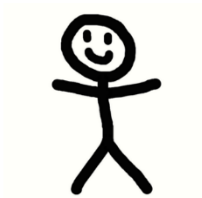

Up-front fee $p_1$ + #units · (per-unit fee $p_2$)

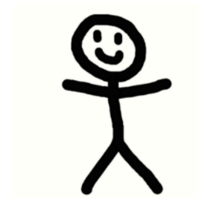
# Main contributions

- First known efficient algorithm for learning revenue-maximizing two-part tariffs.
  - Mechanism designer has access to samples from the distribution over buyers' values rather than an explicit description thereof.

# How to choose prices?

- Seller has $K$ units to sell, wants to maximize revenue
- Buyer has values $v(1), v(2), \dots, v(K)$





Seller: sets two-part tariff $(p_1, p_2)$

Seller's revenue: $p_1 + q \cdot p_2$

Buyer: purchases $q$ units maximizing

$$v(q) - (p_1 + q \cdot p_2)$$

Challenge: Seller does not see $v(1), \dots, v(K)$. Buyer's values are drawn from an unknown distribution from which seller sees samples.

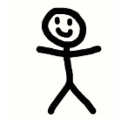
*Goal: efficiently find $(p_1, p_2)$ that yields close-to-optimal expected revenue.*

Instance of *sample-based automated mechanism design* [Sandholm and Likhodedov '04, '05, '15]

# Empirical revenue maximization

Seller sees $N$ IID samples from unknown distribution $D$.

Chooses $(p_1, p_2)$ to maximize empirical revenue:



$v^1(1), \ldots, v^1(K) \sim D$

$\vdots$

$v^N(1), \ldots, v^N(K) \sim D$

$$\widehat{Rev}_S(\boldsymbol{p}) = \frac{1}{N} \sum_{i=1}^{N} Rev_{v^i}(\boldsymbol{p})$$

[Balcan, Sandholm, Vitercik '18] $N = O(\log K)$ samples suffice to guarantee that with high probability, the empirical-revenue-maximizing two-part tariff is near-optimal on a new buyer.

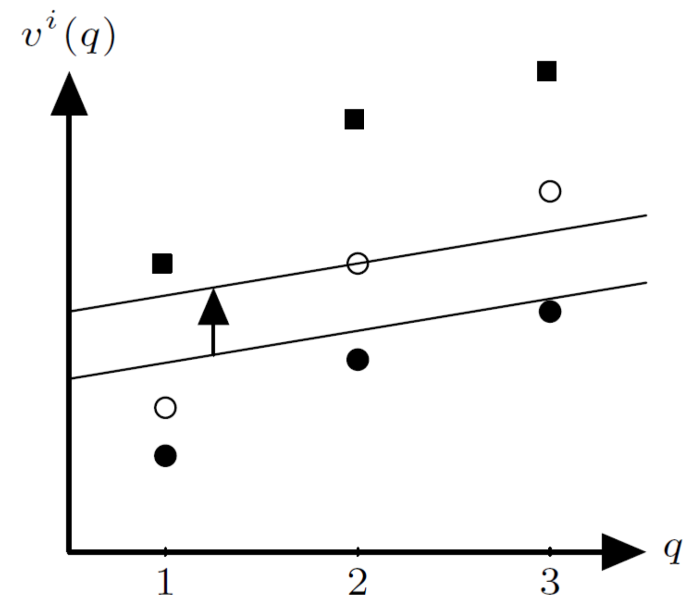
# Algorithm for a single two-part tariff

**Theorem:** *The empirical-revenue maximizing single tariff over a sample set of size $N$ can be found in $O(N^3K^3)$ time.*

- Space of two-part tariffs $(p_1, p_2)$ is infinite, but two crucial insights reduce search space to $N^2K^2$ two-part tariffs.

(1) If $(p_1, p_2)$ is a two-part tariff that maximizes empirical revenue over $S$, the line with $y$-intercept $p_1$ and slope $p_2$ passes through a point $(q, v^i(q))$ for some quantity $q$ and sample $i$.

This is because a vertical shift of a two-part tariff until it passes through such a point only increases revenue.



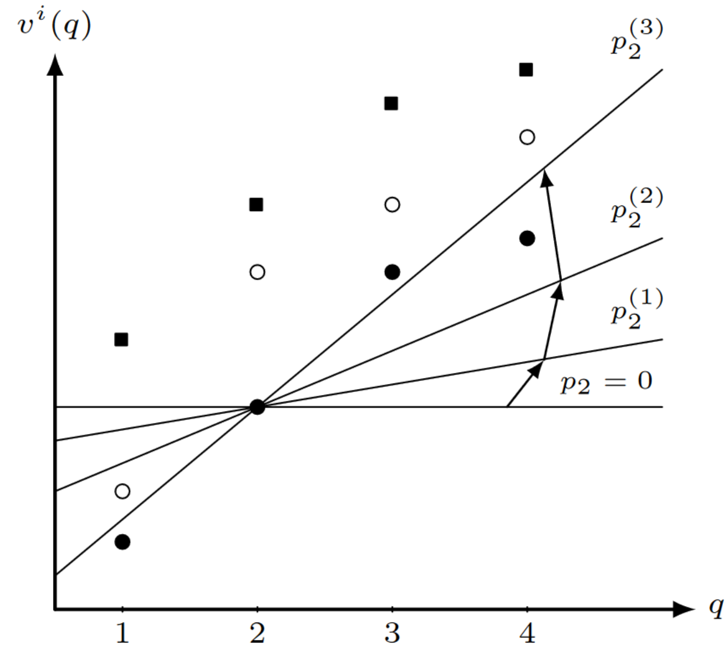
# Algorithm for a single two-part tariff (cont.)

**Theorem:** *The empirical-revenue maximizing single tariff over a sample set of size* $N$ *can be found in* $O(N^3K^3)$ *time.*

- Space of two-part tariffs $(p_1, p_2)$ is infinite, but two crucial insights reduce search space to $N^2K^2$ two-part tariffs.

(2) Hinge a two-part tariff at $(q, v^i(q))$ and continuously rotate it, starting with slope $p_2 = 0$. Each buyer's most-preferred quantity changes at most $K$ times, with revenue varying linearly between each change.

Can compute the "change" points. $NK$ "change" points for each of the $NK$ "hinge" points -> $N^2K^2$ two-part tariffs to search over.



# Menus of two-part tariffs

- Seller can offer a *menu* $(p_1^1, p_2^1), \dots, (p_1^L, p_2^L)$ of $L$ two-part tariffs to extract more revenue
- Buyer with values $v(1), \dots, v(K)$ purchases quantity $q$ of two-part tariff $r$ to maximize
$$v(q) - (p_1^r + q \cdot p_2^r)$$

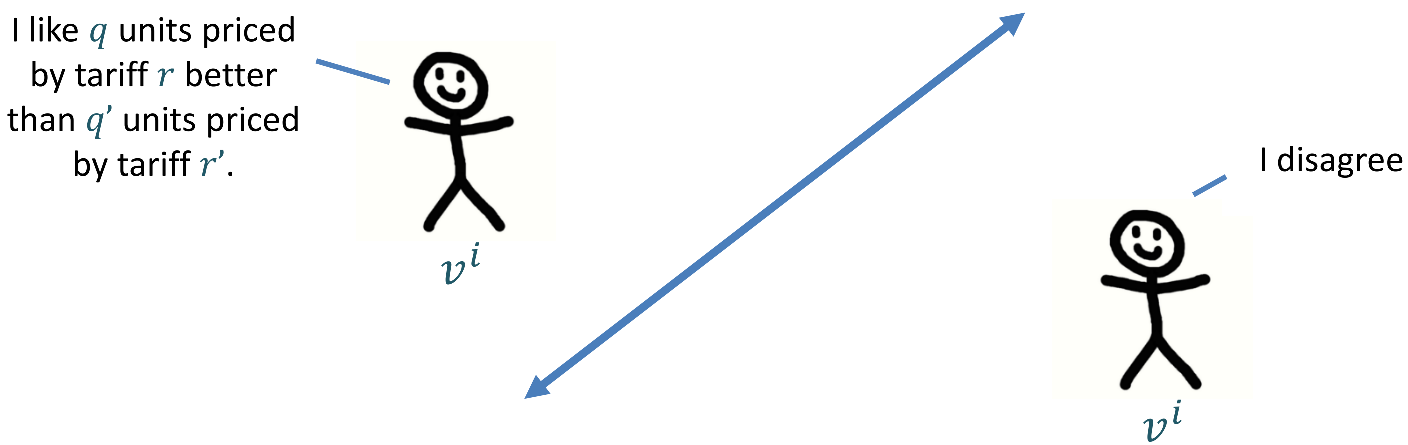
[Balcan, Sandholm, Vitercik '18] $N = O(L \log(K))$ samples suffice to guarantee that with high probability, the empirical-revenue-maximizing menu of two-part tariffs is near-optimal on a new buyer.

This paper: an $(NK)^{O(L)}$-time algorithm to compute the empirical-revenue-maximizing menu of two-part tariffs.

# Algorithm for a menu of two-part tariffs

- $N(KL)^2$ hyperplanes partitioning $\boldsymbol{R}^{2L}$ such that empirical revenue is linear over each region determined by the hyperplanes:

$$v^i(q) - (p_1^r + q \cdot p_2^r) = v^i(q') - \left(p_1^{r'} + q \cdot p_2^{r'}\right)$$




for each sample $i = 1, \ldots, N$, each pair of quantities $q, q'$, and each pair of two-part tariffs $r, r'$.

# Algorithm for a menu of two-part tariffs (cont.)

- $N(KL)^2$ hyperplanes partition $\boldsymbol{R}^{2L}$ into at most $N^{2L}K^{4L}$ convex polytopes.
- The empirical-revenue maximizing menu of two-part tariffs *within a single region* is the solution to a linear program with $2L$ variables and $N(KL)2$ constraints.
- There is a simple $poly(|H|)$-time algorithm that outputs representations of each convex polytope as a set of LP constraints.

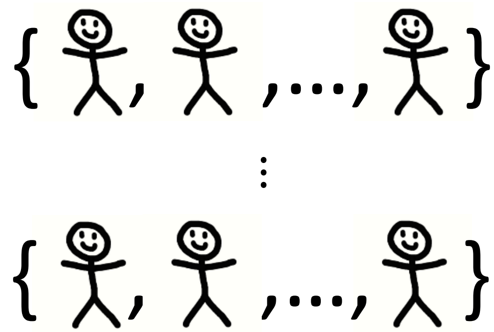
**Theorem:** *The empirical-revenue maximizing menu of $L$ two-part tariffs over a sample set of size $N$ can be found in $(NK)^{O(L)}$ time.*

In most applications, $L$ is a small constant, typically $2$ or $3$.

# Multiple buyers

Selling to $n$ buyers.

See $N$ samples, each consisting of $n$ buyers.



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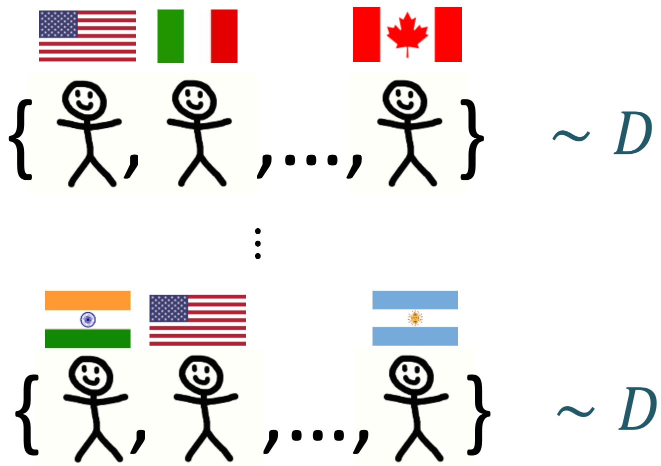
Both algorithms generalize:

- Optimal single two-part tariff: $O(n^3N^3K^3)$-time algorithm.
- Optimal menu of two-part tariffs: $(nNK)^{O(L)}$-time algorithm.

# Market segmentation

- Each buyer belongs to one of $M$ markets
- Seller can set $M$ different menus, one for each market




[Balcan, Sandholm, Vitercik ‘18] $N = O(ML \log(nK))$ samples suffice to guarantee that with high probability, the empirical-revenue-maximizing two-part tariff structure is near-optimal on a new buyer.

# Market segmentation (feasibility)

- Seller must ensure that the total demand from each market does not exceed capacity $K$.

**Theorem:** *Let* $N \geq O_\varepsilon(ML \log(nK))$. *With high probability over the draw of* $N$ *samples, if* $\boldsymbol{p}_1, \ldots, \boldsymbol{p}_M$ *are two-part tariff menus that are feasible over the samples,*

$$\Pr_{v \sim D}(\boldsymbol{p}_1, \ldots, \boldsymbol{p}_M \text{ are feasible for } v) \geq 1 - \varepsilon.$$

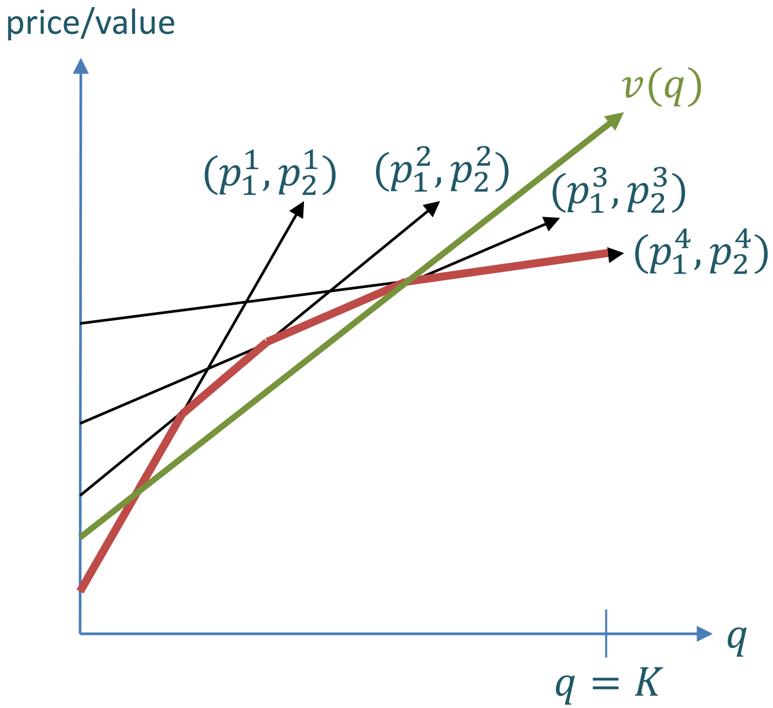
# Market segmentation (computational hardness)

- Very simple case: buyers have additive valuations, i.e.

$$v(q_1 + q_2) = v(q_1) + v(q_2)$$

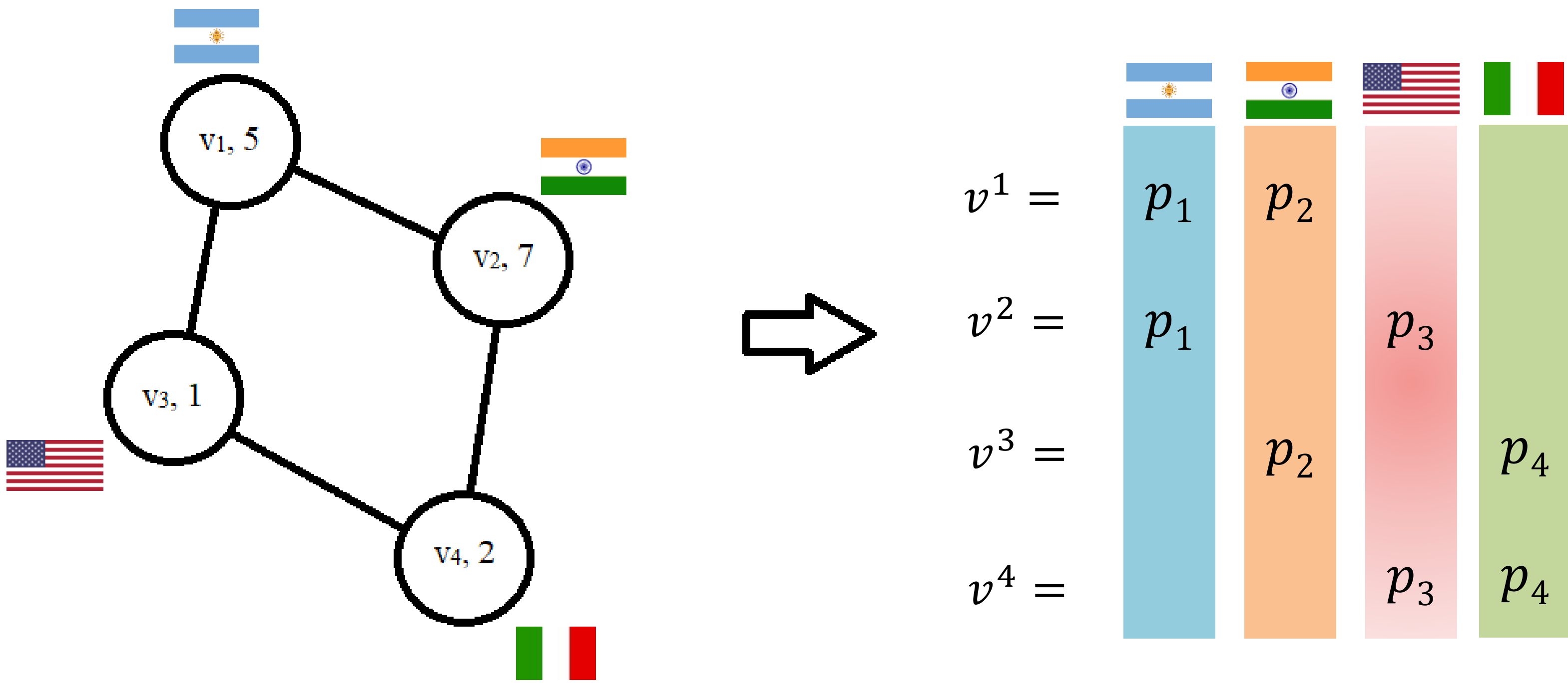
- "Effective price curve" is concave and increasing -> All buyers want the full $K$ units (or nothing).

- Parameters $L$ and $K$ vanish, seller simply needs to choose a single price for each market.




**Theorem:** *Finding the empirically-optimal collection of market prices is NP-hard.*

# Market segmentation (computational hardness)




- Edges -> samples, vertices -> markets.
- $p_i = \frac{w_i|E|}{\deg(v_i)}$ is how much buyers in market $i$ value the bundle.
- Independent set of vertices (markets) corresponds to setting prices such that only buyers in those markets will purchase the bundle. (Markets connected by an edge -> cannot simultaneously purchase the bundle.)

# Market segmentation (IID buyers)

- If each buyer (across markets) is drawn independently from the same distribution, can compute the non-market-segmented solution and reuse it in each market.

- Reusing the non-market-segmented solution is nearly as good as the optimal market-segmented solution (with high probability).

- Can circumvent computational hardness and use either the single-tariff algorithm or the algorithm for a menu of two-part tariffs.

# Conclusion

- Algorithms for learning revenue-maximizing two-part tariff structures from buyer valuation data.
- Polynomial-time algorithm for computing optimal single two-part tariff.
- Algorithm for computing optimal menu of two-part tariffs with runtime polynomial in all parameters but menu length.
- Market segmentation: problem is NP-hard even in simple settings. Assuming IID bidders circumvents this hardness.