Maximizing Revenue Under Market Shrinkage and Market Uncertainty

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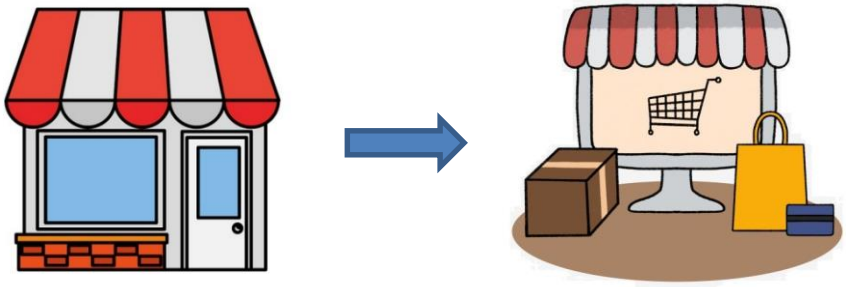
Examples of shrinking markets

Cord cutters

Retail stores



Labor markets among a shrinking population

www.ere.net › labor-market-where-is-everybody-the-sh...

Labor Market: Where Is Everybody? (The Shrinking Labor ...

Sep 24, 2020 — Simply put, the **labor force** participation rate has been falling. The rate for men has been trending downward for nearly 60 years, from 86.7% in ...

www.epi.org › news › shrinking-labor-force-explains-d...

Shrinking labor force explains drop in unemployment

In her analysis of the report, labor economist Heidi Shierholz explained that most of that decline can be explained by the drop in the **labor force** participation rate ...

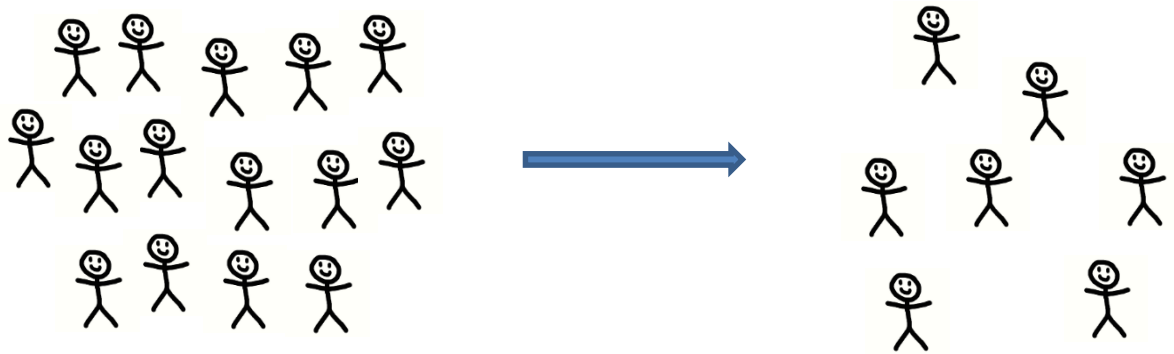
www.wsj.com › articles › covid-shrinks-the-labor-market-...

Covid Shrinks the Labor Market, Pushing Out Women and ...

Dec 3, 2020 — Nearly four million Americans have stopped working or looking for jobs, a 2.2% contraction of the U.S. work force. A smaller **labor market** leaves ...

Modeling a shrinking market

- Fixed set $S = \{v_1, \ldots, v_n\}$ of bidder valuations
- Seller knows S
- Each bidder in S shows up independently with probability p



S $\qquad$ S_0

What fraction of revenue can the seller guarantee?

$$\sup_M \mathbf{E}[\mathrm{Rev}_M(S_0)] \geq (???) \cdot W(S)$$

Revenue loss can be drastic

- At first glance answer might appear to be p (or even higher, if revenue thought to have diminishing returns in number of buyers)

- Example 1: $\mathbf{E}[\mathrm{Rev}_{VCG}(S_0)] = p^2 \mathrm{Rev}_{VCG}(S) = p^2(W(S) - \varepsilon)$
  - Due to reduced competition among buyers

c	0	0
$c - \varepsilon/m$	0	0
0	c	0
0	$c - \varepsilon/m$	0
0	0	c
0	0	$c - \varepsilon/m$

VCG gets payment of $c - \varepsilon/m$ for each item so $\mathrm{Rev}_{VCG}(S) = mc - \varepsilon = W(S) - \varepsilon$

But

$$\mathbf{E}[\mathrm{Rev}_{VCG}(S_0)] = \sum_{\text{item } i} \mathbf{E}[\text{Rev from item } i]$$

$$= p^2(mc - \varepsilon)$$

Revenue loss can be drastic

If valuations can depend on what other bidders receive, things are even worse

Theorem (Balcan, **Prasad**, Sandholm NeurIPS'22). For any $\varepsilon > 0$ there exists a set S of bidders with allocational valuations such that

$$\sup \mathbf{E}[\mathrm{Rev}_M(S_0)] \leq p^{m/2} \cdot (\mathrm{Rev}_{VCG}(S) + 2\varepsilon) + \varepsilon$$

where the supremum is over all possible auctions M.

Escaping large revenue loss

Enabled by two main assumptions:

- *Winner monotonicity*
  - if bidder i wins in VCG, and j leaves, i still wins in VCG
- *Welfare submodularity*
  - efficient welfare a submodular function

e.g. bidders with gross-substitutes valuations

How much revenue can be preserved?

General possibility result: rich enough set of mechanisms always contains one robust to shrinkage

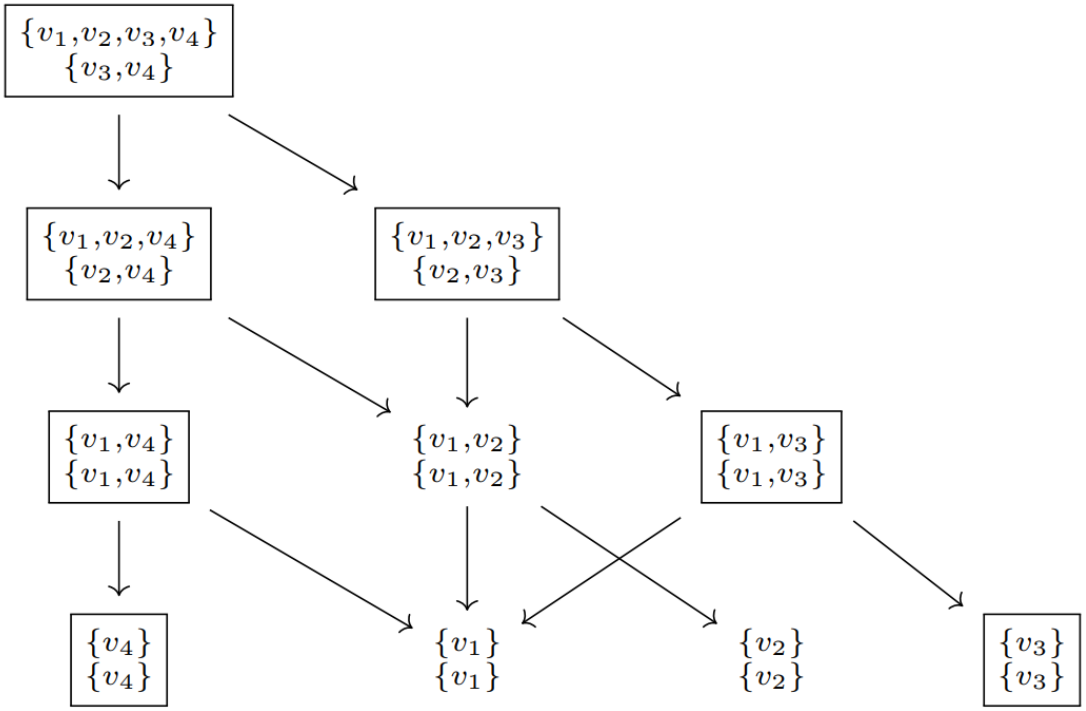
Theorem (Balcan, **Prasad**, Sandholm NeurIPS'22). Exists auction M s.t.

$$\mathbf{E}[\mathrm{Rev}_M(S_0)] \geq \Omega\left(\frac{p^2}{k^{1+\log_{1/\gamma}(4/p)}}\right) \cdot W(S)$$

γ a constant depending on S, $k \approx$ max number of winners in VCG

A shrinkage-robust auction can be computed by sampling simulated shrunken markets and maximizing empirical revenue

Techniques



- *Winner diagram:* concise way of capturing all meaningful executions of an auction
- Randomize over a high-welfare subgraph of the winner diagram

Practically-motivated applications

- Our result yields refined guarantees when the mechanism designer:
  - Limits the number of winners
  - Places bundling constraints on the items

Conclusions

- First formal model of market shrinkage in combinatorial auctions
- Can serve as a testbed for many other mechanism design questions with market uncertainty