Learning Within an Instance for Designing High-Revenue Combinatorial Auctions

Nina Balcan[1], Siddharth Prasad[1], Tuomas Sandholm[1, 2, 3, 4]

[1] Carnegie Mellon University
[2] Optimized Markets, Inc.
[3] Strategic Machine, Inc.
[4] Strategy Robot, Inc.

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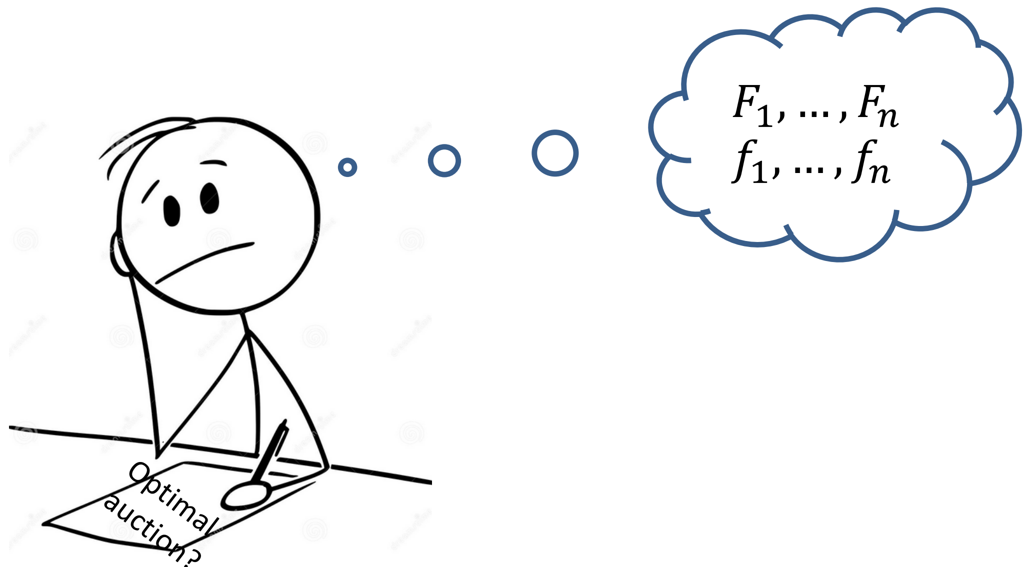
Background and motivation

- Classical mechanism design
  - 1981: Myerson showed how to sell a single item to maximize revenue (uses details of the distribution of buyers' values for the item)
  - Today: don't know how to sell two items optimally

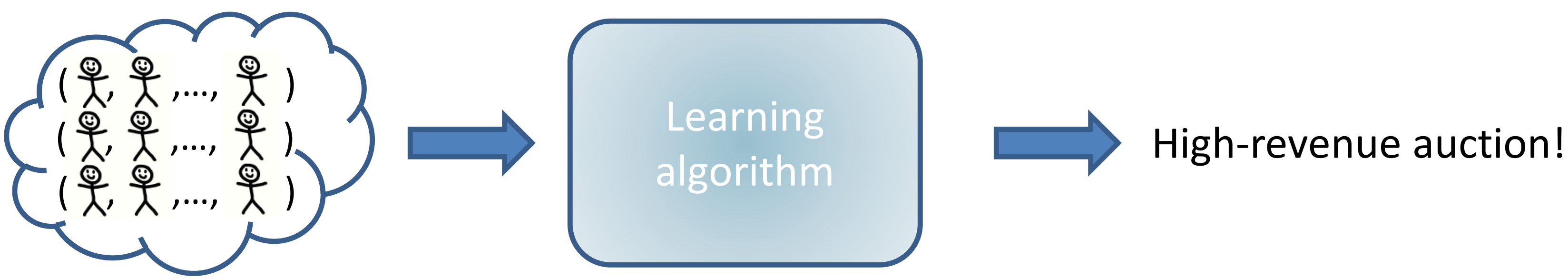


Background and motivation

- Automated mechanism design [Conitzer and Sandholm UAI '03]:
  - why struggle with the hard economics problem of designing explicit mechanisms when a computer program can do it for you?
  - requiring details of distributions -> computational hardness in many settings

Background and motivation

- Sample-based automated mechanism design
  - use machine learning, don't need details about distribution [Balcan, Blum, and Mansour FOCS'05, Sandholm and Likhodedov AAAI '04, '05, Operations Research '15, Mohri and Medina ICML '14, Morgenstern and Roughgarden NIPS '15, Balcan, Sandholm, and Vitercik EC'18, Duetting et al. ICML'19, Balcan, **Prasad**, and Sandholm IJCAI'20]

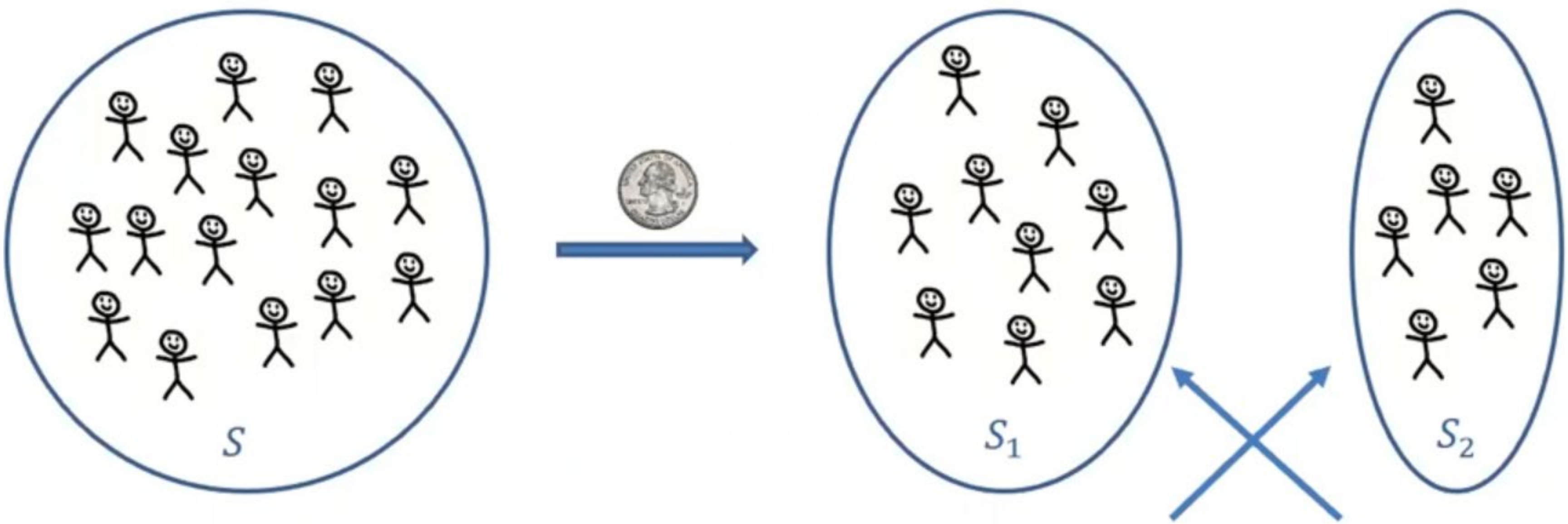


Setup and background

- We know how to learn high-revenue auctions for limited supply across instances/samples [Mohri and Medina ICML '14, Morgenstern and Roughgarden NIPS '15, Balcan, Sandholm, and Vitercik EC'18]
- What if the mechanism designer does not even have access to samples?
  - Set S of bidders shows up. How to design an auction that performs well on S with no prior information about S?
- Balcan, Blum, Hartline, Mansour FOCS'05 answer this in the *unlimited-supply* setting.
  - Random-sampling mechanism, can be viewed as "learning within an instance"
  - Several follow up works on random sampling for unlimited supply

Random-sampling mechanisms

Balcan, Blum, Hartline, Mansour (FOCS 2005)



Revenue = $\mathrm{Rev}_{M_1}(S_2) + \mathrm{Rev}_{M_2}(S_1)$

M_1 optimal mechanism for S_1

M_2 optimal mechanism for S_2

- Incentive compatibility: mechanism used on a bidder's group independent of report
- Sample complexity guarantee: as number of bidders grows, random sampling achieves a (1-ε)-approximation to the optimum.
- **Crucially relies on lack of supply constraints.**

Combinatorial auctions crash course

- Seller has m indivisible items to sell among set S of n bidders.
- Bidders have combinatorial valuations $v_i: 2^{\{1,\dots,m\}} \to \mathbf{R}_{\geq 0}$.
- For reported valuations $v_1, \dots, v_n$, an auction M specifies an allocation $\alpha(v_1, \dots, v_n)$ and payments $p_i(v_1, \dots, v_n)$.
- Seller wants to design M that extracts high revenue in an incentive compatible manner, that is,

$$v_i\big(\alpha(v_1, \dots, v_n)\big) - p_i(v_1, \dots, v_n) \geq v_i\big(\alpha(v_1, \dots, \hat{v}_i, \dots, v_n)\big) - p_i(v_1, \dots, \hat{v}_i, \dots, v_n)$$

Combinatorial auctions crash course

Vickrey-Clarke-Groves (VCG) auction:

- use allocation α^* that maximizes welfare

$$W(\alpha) = \sum_{i=1}^{n} v_i(\alpha)$$

- bidder i pays

$$\max_{\alpha} \sum_{j \neq i} v_j(\alpha) - \sum_{j \neq i} v_j(\alpha^*)$$

- VCG is incentive compatible

Combinatorial auctions crash course

λ-auction: parameterized by $\lambda \in \mathbf{R}^{(n+1)^m}$

- use allocation α^* that maximizes welfare plus boost

$$\sum_{i=1}^{n} v_i(\alpha) + \lambda(\alpha)$$

- Bidder i pays

$$\max_{\alpha}\left[\sum_{j \neq i} v_j(\alpha) + \lambda(\alpha)\right] - \left[\sum_{j \neq i} v_j(\alpha^*) + \lambda(\alpha^*)\right]$$

- λ-auctions are incentive compatible

- Many other parameterized generalizations of VCG exist

Revenue benchmark

- We compare all our revenue guarantees to the very strong benchmark of

$$W(S) = \max_{\alpha} W(\alpha)$$

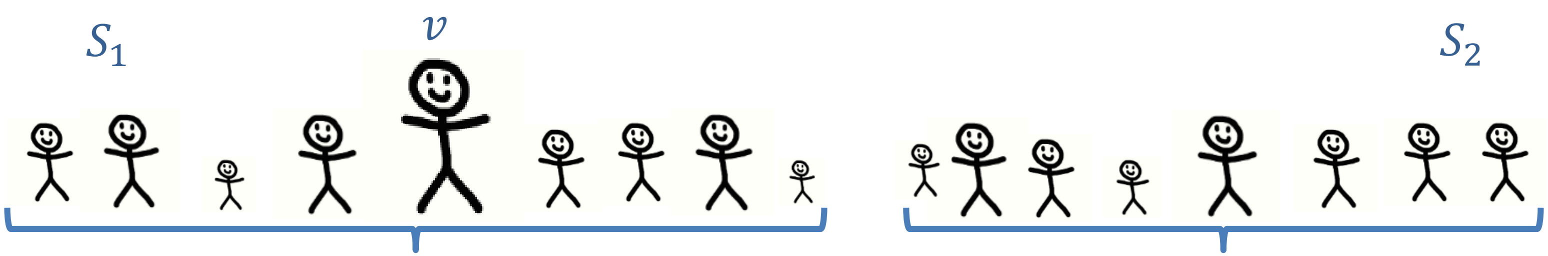
- No auction can obtain revenue greater than $W(S)$, since no bidder would pay more than her value for the bundle she receives.

Difficulties of limited supply

- Allocations must be feasible.
- Balcan, Blum, Hartline, Mansour set optimal prices for items, and let buyers purchase according to their demand functions.
- Not possible for limited supply without considering complicated benchmarks depending on the order of buyers purchasing, etc.
- We apply variants of λ-auctions that prescribe feasible allocations.

Difficulties of limited supply

- Try to adapt random sampling to limited supply
- Attempt 1: Partition S into S_1, S_2 as before, compute optimal mechanism for S_1, run on S_2
  - Single item, sold via second-price auction with reserve price
  - Item goes to highest bidder v_1, payment = $\max\{v_2, r\}$



$r_{OPT} = v$ $\mathrm{Rev}_{r_{OPT}}(S_1) = v$ $Rev_{r_{OPT}}(S_2) = 0$

With probability ½, revenue = 0!

Difficulties of limited supply

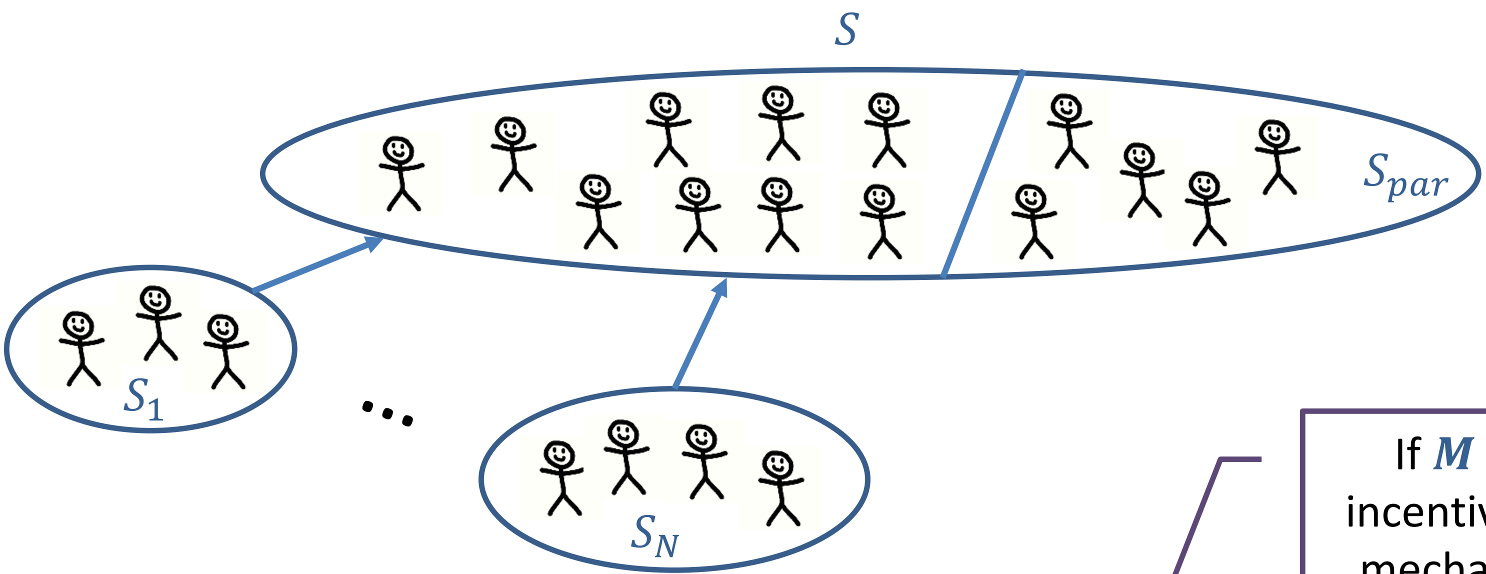
- Try to adapt random sampling to limited supply

- Attempt 2: Partition S into $S_1,\ldots,S_N,S_{N+1}$. Compute empirically optimal mechanism for $S_1,\ldots,S_N$, run on S_{N+1}

  - If auction class is complex, N needs to be large for generalization guarantees to hold

  - S_{N+1} contains a tiny fraction of bidders, losing a lot of revenue

Learning Within an Instance (LWI) mechanism



If $\boldsymbol{M}$ is a class of incentive compatible mechanisms, LWI is incentive compatible

1. Sample participatory group $S_{par} \sim_p S$
2. Sample learning groups $S_1, \ldots, S_N \sim_q S \setminus S_{par}$
3. Compute ERM mechanism $\widehat{M} \in \boldsymbol{M}$ over learning groups
4. Run mechanism $\widehat{M}$ on S_{par}

Basic general guarantee

Theorem (Balcan, **P.**, Sandholm). For $N \geq N_{\boldsymbol{M}}(\varepsilon, \delta)$,

$$Rev_{\widehat{M}}(S_{par}) \geq W(S)\left(L_{\boldsymbol{M}}(S_{par}) - \varepsilon\right) - 2\tau_{\boldsymbol{M}}(q, S_{par})$$

with probability $\geq 1 - 2\delta$ over the draw of the learning groups.

- $N_{\boldsymbol{M}}(\varepsilon, \delta) = O\left(\frac{\mathrm{Pdim}(\boldsymbol{M}) \ln\left(\frac{1}{\delta}\right)}{\varepsilon^2}\right)$ a standard sample complexity term
- $L_{\boldsymbol{M}}(S_{par}) = \frac{OPT_{\boldsymbol{M}}(S_{par})}{W(S)}$ measures how much revenue is lost by restricting to S_{par}
- $\tau_{\boldsymbol{M}}(q, S_{par})$ a complexity/uniformity measure we call *partition discrepancy*

$$\tau_{\boldsymbol{M}}(q, S_{par}) = \sup_{M \in \boldsymbol{M}} \left| Rev_M(S_{par}) - \mathbf{E}_{S_0 \sim_q S \setminus S_{par}}[Rev_M(S_0)] \right|$$

Making the guarantee more concrete

Bounding partition discrepancy

Theorem (Balcan, **P.**, Sandholm). For $N \geq N_{\boldsymbol{M}}(\varepsilon, \delta), p = 1/2, q = 1/3,$ and $W(S)$ sufficiently large,

$$Rev_{\widehat{M}}\left(S_{par}\right) \geq W(S)\left(L_{\boldsymbol{M}}(S_{par}) - 2\varepsilon\right)$$

with probability $\geq 1 - 3\delta$ over the run of LWI.

Instantiation for specific auction class

Theorem (Balcan, **P.**, Sandholm)**.** For $N \geq N_{\boldsymbol{M}}(\varepsilon, \delta), W(S)$ sufficiently large, and one "niceness" condition,

$$Rev_{\widehat{M}}\left(S_{par}\right) \geq W(S)\left((1-\eta)p - \varepsilon\right) - 2\tau_{\boldsymbol{M}}\left(q, S_{par}\right)$$

with probability $\geq 1 - 2\delta$ over the draw of the learning groups. $\boldsymbol{M}$ is the class of "bundling-boosted auctions".

Comparison to existing results

- Subadditive valuations
  - $O(2^{\sqrt{\log m \log\log m}})$-approximation [Balcan, Blum, Mansour EC 2008]
  - $O(\log^2 m)$-approximation [Chakraborty, Huang, Khanna FOCS 2009; SICOMP 2013]
- Additive valuations
  - $O(\log(h/l))$-approximation, h, l highest and lowest values for any bundle [Sandholm and Likhodedov AAAI 2005; Operations Research 2015]
- Our guarantees
  - No assumptions on valuation functions
  - Fine tuned to structure in the set of bidders

Partition discrepancy

$$\tau_{\boldsymbol{M}}(q, S_{par}) = \sup_{M \in \boldsymbol{M}} |Rev_M(S_{par}) - \mathbf{E}_{S_0 \sim_q S \setminus S_{par}}[Rev_M(S_0)]\,|$$

If S is a replica economy (consists of copies of the same ground set of bidders), then $\tau_{\boldsymbol{M}}(1, S_{par}) = 0$ with high probability over the draw of $S_{par} \sim_{1/2} S$.

Theorem (Balcan, **P.**, Sandholm). With probably $\geq 1 - \delta$ over the draw of $S_{par} \sim_{1/3} S$**,**

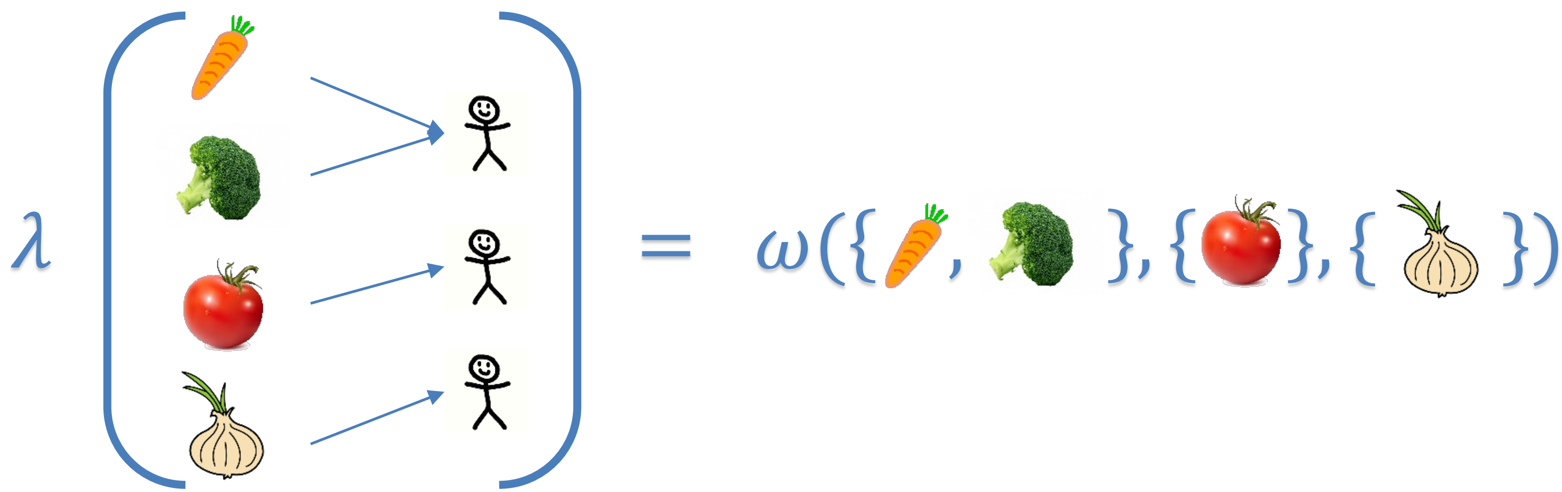
$$\tau_{\boldsymbol{M}}(q, S_{par}) \leq O\left(\|\widetilde{\boldsymbol{v}}\|_2 \sqrt{n\mathrm{Pdim}(\boldsymbol{M}) \ln\left(\frac{\mathrm{Pdim}(\boldsymbol{M}) \cdot W(S)}{\delta}\right)}\right)$$

Proof leverages covering number bounds from learning theory.

Population-size-independent auctions

- New auction classes that can be described independent of the number of bidders -> Sample complexity of LWI independent of number of bidders!

- Bundling-boosted auctions: prescribes boosts for bundlings
  e.g. $\omega(\{1,2\},\{3,4\}) = 5, \omega(\{1\},\{2,3\},\{4\}) = 2, \omega(\{1,2,3,4\}) = 10.$
  - Allocational boost is ω(*coarsest* bundling the allocation respects)



Population-size-independent auctions

Theorem (Balcan, **P.**, Sandholm). The pseudodimension of the class of bundling-boosted auctions run on m items and (up to) n bidders is $O(m^m)$.

- We show that the class of bundling-boosted auctions contains many high-revenue auctions.
- Under a niceness condition, basically $p \cdot W(S)$ revenue can be obtained.
- We introduce other population-size-independent auctions that are subclasses of well known and extensively studied auction classes.

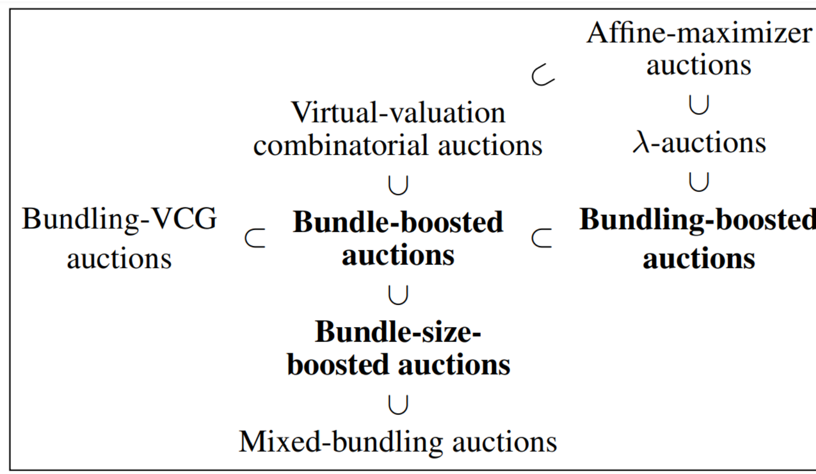


Figure 1: Containment relations between auction classes. New auction classes introduced in this paper are in boldface.

Efficient Learning Within an Instance

- For subclasses that boost a constant number of bundlings, we show how LWI can be implemented efficiently by leveraging practically-efficient routines for winner determination.

Theorem (Balcan, **P.**, Sandholm). The pseudodimension of the class of Φ-bundling-boosted auctions run on m items and (up to) n bidders is $O(|\Phi| \log m|\Phi|)$.

Theorem (Balcan, **P.**, Sandholm). Empirical revenue maximization over Φ-bundling-boosted auctions can be computed in $(Nm|\Phi|)^{O(|\Phi|)} + 2w(m,n)Nm|\Phi|$ time, where $w(m,n)$ is the time required to solve winner determination for n buyers and m items.

Structural revenue maximization

- Which auction class to use with LWI to prevent overfitting?
- Our guarantees provide a regularization term to use instead of vanilla ERM that optimizes the tradeoff between high training revenue and overfitting.

Conclusion

- New mechanism that learns within an instance to design high-revenue combinatorial auction for limited supply
- Generalizes and adapts previous random sampling approaches that only worked for unlimited supply
- We prove extremely general guarantees that require no assumptions on valuation functions
  - Introduced complexity/uniformity measure called partition discrepancy
  - Introduced new population-size-independent auctions
- Gave an algorithm for efficiently running our mechanism